HEAT TRANSFER FLOW IN A CHANNEL BOUNDED BY STRETCHING WALLS WITH HALL CURRENTS, THERMAL RADIATION, DISSIPATION AND NON-UNIFORM HEAT SOURCES

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Received: Apr. 2019 Accepted: May. 2019 Published: Jun. 2019

Abstract: We discuss he effect of thermal radiation and Hall currents on convective heat transfer flow of an electrically conducting, viscous fluid in a vertical channel bounded by stretching walls in the presence of non-uniform heat sources. The non-linear equations have been have solved by Runge-Kutta method along Shooting technique. It is found that an increase in Hall Parameter(m) reduces the skin friction and enhances the Nusselt number at the walls. Higher the thermal radiation smaller Nu at the left wall and larger at the right wall. An increase in Ec increases Nu at $\eta=\pm1$.

Keywords: Thermal Radiation, Hall Effects, Dissipation, Non-Uniform Heat Sources, Stretching Walls.

1. Introduction: Laminar flows through channels have applications in the fields of gas diffusion, ablation cooling, filtration, microfluidic devices, surface sublimation, grain regression(as in the case of combustion in rocket motors) and the modelling of air circulation in the respiratory system. Laminar air –flow systems have been used by the aerospace industry to control particular contamination. The laminar flow cabinet have been used in the maintenance of negative pressure and in the adjustment of the fans to exhaust more air. Therefore, the Navier-Stokes equations which are the governing equations for these problems have attracted the interest of the researchers. Sutton and Barto(16) described an exact solution of Navier-Stokes equations for motion of an incompressible viscous fluid in a channel with different pressure gradients .Their solutions are helpful in verifying and validating computational models of complex on unsteady motions,to guide the design of fuel injectors and controlled experiments. Simulation of flow through microchannels with design roughness was presented numerically by Rawool et al(11) A numerical investigations is made by Robinson(12) for the problem of steady laminar incompressible flow in a porous channel with uniform suction at both walls .Taylor et al(17) studied three dimensional flow buy uniform suction through parallel porous walls. The investigations of Taylor(17) were further extended to a more general three dimensional stagnation point which can capture the phenomena in a single class of state by Hewitt et al(7).Two dimensional viscous incompressible fluid flow between two porous walls with uniform suction was analysed by Cox(4).Berman(2) proposed the two dimensional laminar steady flow through a porous channel which was driven by suction or injection .Similarity one/two dimensional laminar flow in a porous channel with wall suction or injection was examined analytically by Laurent et al(10).The problem of fluid flow in a channel with porous walls was solved by Karode(8). The exact solution for two dimensional steady laminar flow through a porous channel was generalized by Terril(18),Shrestril and Terril(14,15), Brady(3), Waston et al(20) and Cox(4) under varied conditions. Deng and Marinez(5) worked on two dimensional flow of a viscous fluid in a channel partially filled with porous medium with wall suction. Wang(19) worked on viscous flow due to stretching sheet with slip and suction and proved a closed form unique solution for two dimensinonal flows. For asymmetric stretching both existence and uniqueness were shown. Muhammad Asraf et al(1) investigated micropolar fluid flow in a channel with Shrinking walls. Hajipour and Dehkordi(6) studied the transient behaviour of fluid flow and heat transfer in vertical channel partially filled with a porous medium including the effects of inertial term and viscous dissipation. Kasif Ali et al(9)have discussed numerical study of micropolar fluid flow and heat transfer in a channel with shrinking and stationary walls. Sarojamma et al(13) have analysed the convective heat transfer flow of a Casson fluid in channel bounded by stretching walls.

In this paper, we investigate the effect of thermal radiation and Hall currents on convective heat transfer flow of an electrically conducting, viscous fluid in a vertical channel bounded by stretching walls. The nonlinear governing equations have been solved by using Runge-Kutta shooting technique. The velocity, temperature have been analysed for different variations of m,A1,B1,Rd,Ec .The rate of Skin friction, rate of heat transfer are evaluated numerically for different variations.

2. Formulation of The Problem:

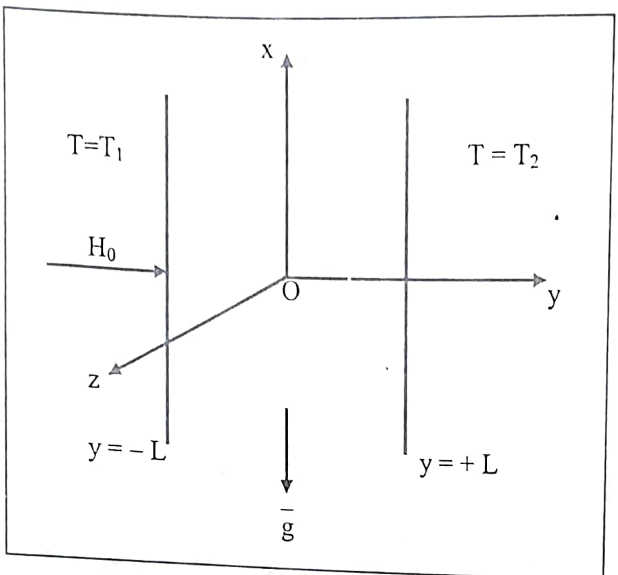

Fig. 1: Configuration of the Problem

The steady two dimensional MHD convective heat and mass transfer flow of a viscous, electrically conducting fluid in a vertical channel bounded by stretcting sheets is considered. We consider a rectangular coordinate system O(x,y.z) with the walls at $y=\pm L$. A uniform magnetic field of strength H_o is applied normal to the walls. Assuming magnetic Reynolds to be small we neglect induced magnetic field in comparison to the applied field. Taking Hall currents into account the equations governing the flow and heat transfer under Boussinesq approximation and Rosseland approximation in the presence of non-uniform heat sources , are

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma\mu_e^2H_o^2}{1+m^2}(u+mw)+\beta g(T-T_0) \quad (2.1)$$

$$u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}=\nu\frac{\partial^2 w}{\partial y^2}-\frac{\sigma\mu_e^2H_o^2}{1+m^2}(mu-w) \quad (2..2)$$

$$\rho C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k_f\frac{\partial^2 T}{\partial y^2}+\left(\frac{k_f us}{x\nu}\right)(A1(T_1-T_2)u+B1(T-T_2))+$$

$$+\frac{16\sigma^{\bullet}T_o^3}{3\beta_R}\frac{\partial^2 T}{\partial y^2}+\frac{\sigma\mu_e^2H_o^2}{1+m^2}(u^2+w^2)+2\mu\left(\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial w}{\partial y}\right)^2\right) \quad (2.3)$$

where A1 and B1 are coefficients of space dependent and temperature dependent internal heat generation or absorption respectively. It is noted that the case A1>o and B1>o, corresponds to internal heat generation and that A1<o and B1<o, the case corresponds to internal heat absorption case. σ* is the

Stephan - Boltzmann constant and β_R is the mean absorption coefficient. T , k_f , Cp , β ,q_r , are defined as temperature of the fluid, thermal diffusivity, specific heat at constant pressure, coefficient of thermal expansion, radiative heat flux respectively.,

The boundary conditions for the velocity, temperature are

$$u(x,-L) = us = bx, u(x,+L) = bx, v(x,\pm L) = 0$$
$$T(x,-L) = T_1, T(x,+L) = T_2 \quad (2.4)$$

Where b>o is the stretching rate of the channel wall,T_1,T_2(with (T_1>T_2)are the fixed temperature of the left and right walls respectively.

Introducing the following Similarity variables

$$\eta = \frac{y}{L}, u = bxf'(\eta), v = -bLf(\eta),$$
$$w = bxg_o(x), \theta = \frac{T - T_2}{T_1 - T_2} \quad (2.5)$$

the non-dimensional governing equations are

$$f^{iv} + \mathrm{Re}_x(f'''f - ff'') - \frac{M}{1+m^2}(f' + mg) + Gr(\theta) = 0 \quad (2.6)$$

$$g'' + \mathrm{Re}_x(fg_o' - f'g_o) - \frac{M}{1+m^2}(mf' - g_o) = 0 \quad (2.7)$$

$$Rd\theta'' + \Pr \mathrm{Re}\,x(f\theta') + \Pr(A_{11}f' + B_{11}\theta) +$$
$$+ \frac{\Pr EcM^2}{1+m^2}(f'^2 + g^2) + \Pr Ec(f'')^2 + (g')^2) = 0 \quad (2.8)$$

Where $Gr = \frac{\beta g(T_1 - T_2)L}{bx}$, is the Grashof number, $M^2 = \frac{\sigma\mu_e^2 H_o^2 L^2}{\nu x}$, magnetic parameter, $R = \frac{2\Omega}{xL}$, Rotation parameter, $\Pr = \frac{\mu C_p}{k_f}$ is Prandtl number, $\mathrm{Re}x = \frac{bL^2}{\mu}$, is the local Reynolds number,

$Ec = \frac{b^2 x^2}{C_p \Delta T}$ is the Eckert number, $m = \omega_e \tau_e$ is the Hall parameter.

Boundary conditions(2.4),in view of equation(2.5) in non-dimensional form reduces to

$$f(\pm 1) = 0, f'(\pm 1) = 1, g_0(\pm 1) = 0, \theta(-1) = 1, \theta(+1) = 0 \quad (2.9)$$

3.Numerical Procedure: The equations(2.6)-(2.8) have been solved by using a shooting technique is consolidated to solve the above system ,which may be a combination of the Runge-Kutta method(to solve first order ODE) and a five dimensional zero discovering algorithm(to locate the missing coordinates). It is note that the missing initial conditions are coupled so the arrangement fulfills the boundary conditions

4. Results And Discussion: In this analysis we investigate the impact of thermal radiation and Hall effects on convective heat and mass transfer flow of a viscous, dissipative fluid in a vertical channel bounded by stretching plates in the presence of non-uniform heat sources. The non-linear governing equations have been solved by utilizing Fourth order Runge-Kutta – Shooting technique.

The primary velocity, secondary velocity, temperature and concentration have been analysed for different variations of m,A11,B11,Rd,Ec We also study Skin friction components(τ_x,τ_z),the rate of heat transfer (Nu), briefly.

1. The variation of primary velocity f ' with Hall parameter(m) shows that f ' enhances in the region (-1.0,0.5) and reduces in the region adjacent to the right wall η=+1.The secondary velocity g and the temperature enhances with increasing m in the entire flow region(figs.2b&2c).The Skin friction component τx reduces at both the walls with m. The component τz enhances at the left wall and reduces at the right wall with incr4easuing m. An increase in m enhances with the rate of heat transfer at both the walls(table.1).
2. Figs.3a-3c represent the primary, secondary and temperature with space dependent heat source(A11).With respect to space dependent heat source /sink parameter (A11>0&A11<0) reduces in the region(-1.0,0.5) while in the region(0.5,1.0),it enhances with A11>0 and reduces with A11<0.The secondary velocities enhances with A11>0 and reduces with A11<0 in the region(-1.0,1.0).The temperature θ reduces in the left half and enhances in the right half of the channel with increasing values of A11>0 while a reversed effect is noticed with that of heat sink(fig.5c). The skin friction components $\tau_x$enhances at η=-1 &+1 while an increase in A11>0 enhances at η=-1 and reduces at =+1. The skin friction $\tau_z$reduces with A11<0 and enhances with A11>0 at both the walls.An increase in A11>0 and A11<0 enhances the rate of heat transfer at η=± 1 (table.1)
3. Figs.4a-4c depict the variation of primary, secondary velocities and temperature with temperature dependent heat sources. In the presence of generating heat sources, the primary velocity reduces in the region(-1.0,0.0) and enhances in the region(0.0,1.0) and in the case of heat absorbing case we notice a reversed effect in the behaviour of f ' in the flow region. The secondary velocity g decreases in the flow region(-1.0,0.0) and enhances in the region(0.0,1.0) with increase in B 11>0 while a reversed behaviour is observed in the case of B11<0(fig.4b).In the presence of heat generating sources, heat is generated which results in an enhancement in the temperature in the entire flow region while the temperature is reduced due to the absorption of energy in the case of heat absorption sources(fig.4c).The skin friction component τx reduces at η=-1 and enhances at η=+|1 with increasing B11<0 and reversed effect is noticed in the behaviour of τx at the walls.τz reduces with B11<0 and enhances with B11>0 at η=±1.The rate of heat transfer enhances at the left wall and reduces at the left wall with B11<0 and enhances at both the walls with B11>0.
4. From the figures (5a-5c) we observed that the primary velocity increases in the flow regions (-1.0,-0.5)&(0.0,0.5) and enhances in the regions(-0.5,0)&(0.5,1.0) with increasing Rd(fig.5a).The secondary velocity enhances in the entire flow region with increasing Rd(fig.5b).The temperature decays in the region(-1.0,-0.5) and grows in the region(-0.5,1.0) with an increase in radiation parameter(Rd)(fig.5c).A rise in Rd enhances τ_x at η=-1 and reduces at η=+1 while τ_z grows at η=±1 with increase in Rd . Also higher the thermal radiation larger Nu at the left wall and larger at the right wall (table.1).
5. From the figures (6a-6c), we observed that the primary velocity reduces in the region(-1.0,-0.5)&(0.0,0.5) and enhances in the regions (-0.,5,0.0)&(0.5,1.0) with increasing values of Eckert number(Ec).The secondary velocity enhances in the entire flow region of the channel with increasing Ec(fig.6b). The temperature enhances with increase in Ec(fig.6c). The Shear stress τ_x reduces at η=+1 and enhances at η= -1 whereas τ_z increases at both the walls η =±1.The Nusselt number increases with Ec at η=-1 and η=+1. (table 1).

5. Conclusions: We aim at investigating the effect of Hall currents, non-uniform heat sources, thermal radiation on heat transfer flow of viscous fluid in a vertical channel bounded by stretching walls under a uniform magnetic field. The highly non-linear fourth order momentum equation, second order equation of energy have been solved by using Runge-Kutta shooting method. The impact of different physical parameters like Hall parameter(m),),Heat source parameters A11,B11,Eckert number(Ec) on the flow characteristics have been studied graphically in detail. An increase in Hall Parameter(m) reduces the skin friction(τx,τz) and enhances the Nusselt number (Nu) at the walls. Higher the thermal radiation smaller Nusseltnumber(Nu) at the left wall and larger at the right wall. An increase in Ec increases Nu at η=±1.

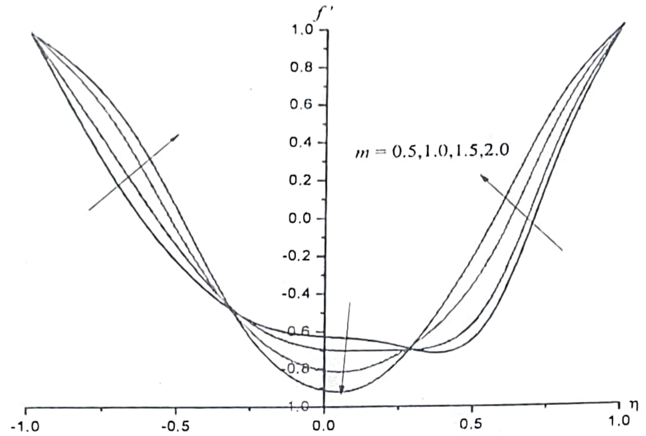

Fig.2a : Variation of f' with m
m=0.5, Rd=0.5, A11=0.5, B11=0.5, Ec=0.01

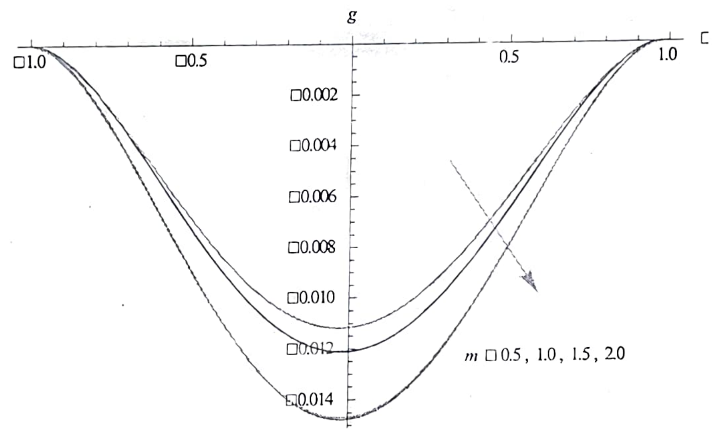

Fig.2b: Variation of g with m
m=0.5, Rd=0.5, A11=0.5, B11=0.5, Ec=0.01

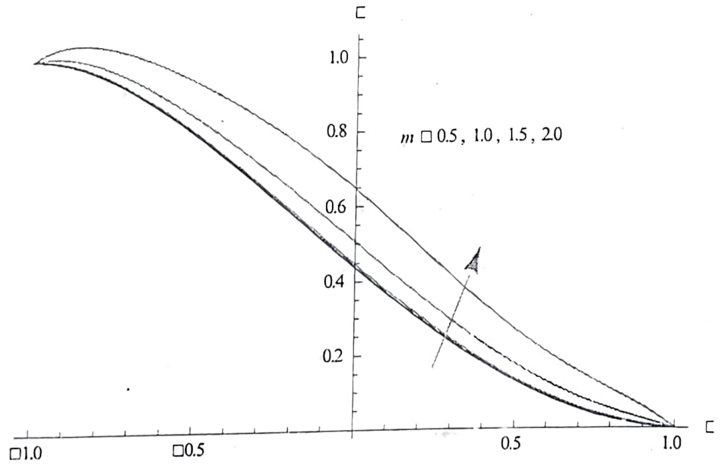

Fig.2c: Variation of θ with m
m=0.5, Rd=0.5, A11=0.5, B11=0.5, Ec=0.01

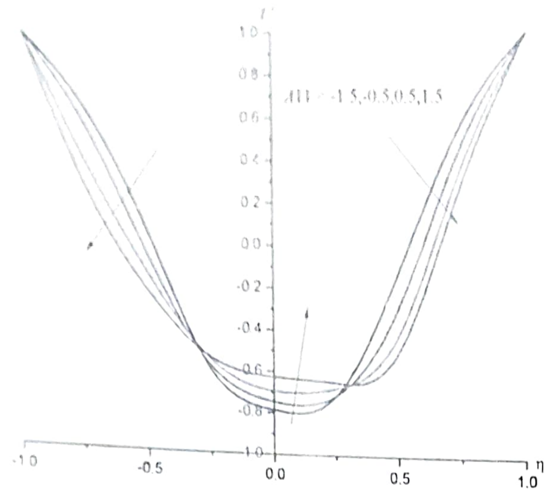

Fig.3a: Variation of f' with A11
m=0.5, Rd=0.5, A11=0.5, B11=0.5, Ec=0.01

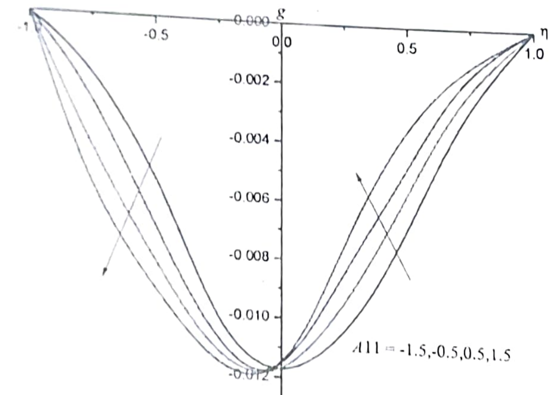

Fig.3b: Variation of g with A11
m=0.5, Rd=0.5, A11=0.5, B11=0.5, Ec=0.01

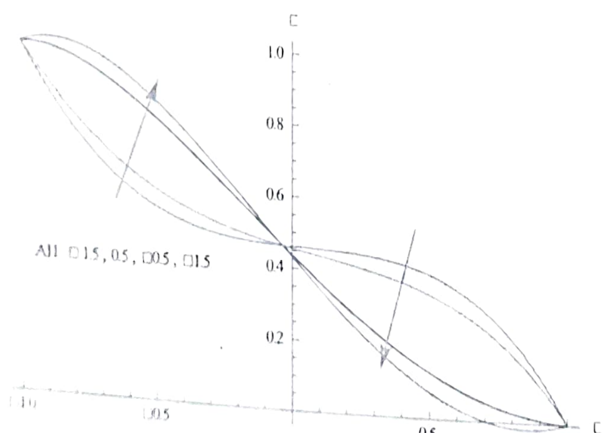

Fig.3c: Variation of θ with A11
m=0.5, Rd=0.5, A11=0.5, B11=0.5, Ec=0.01

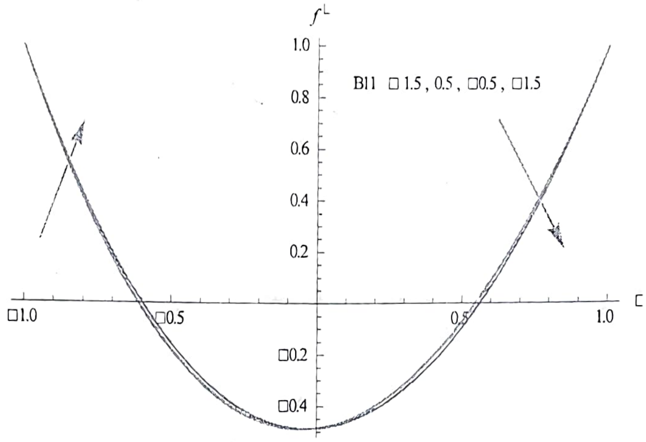

Fig.4a : Variation of f' with B11
m=0.5, Rd=0.5, A11=0.5, B11=0.5, Ec=0.01

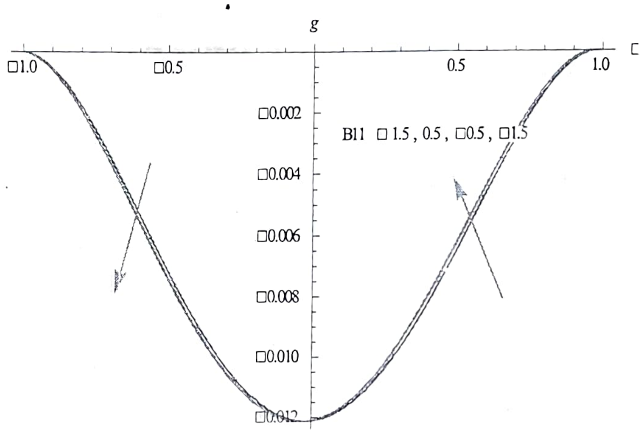

Fig.4b: Variation of g with B11
m=0.5, Rd=0.5, A11=0.5, B11=0.5, Ec=0.01

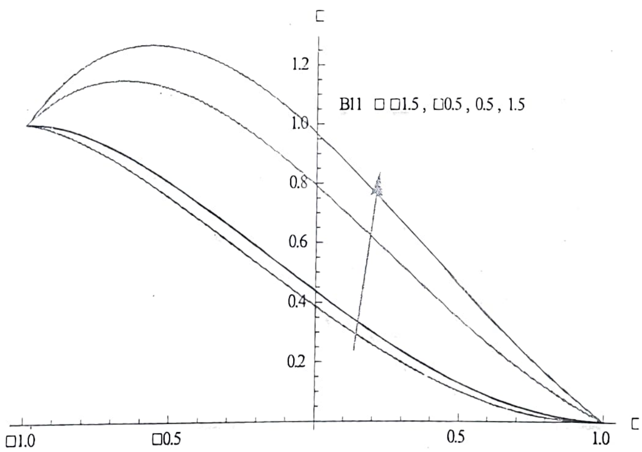

Fig.4c: Variation of θ with B11
m=0.5, Rd=0.5, A11=0.5, B11=0.5, Ec=0.01

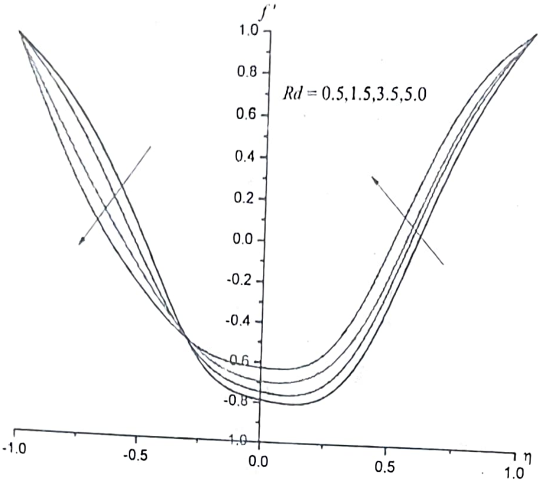

Fig.5a: Variation of f' with Rd
m=0.5, Rd=0.5, A11=0.5, B11=0.5, Ec=0.01

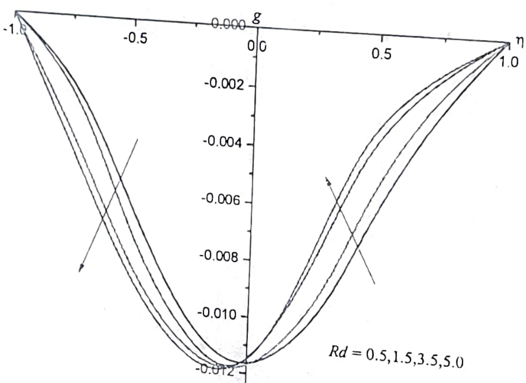

Fig.5b: Variation of g with Rd
m=0.5, Rd=0.5, A11=0.5, B11=0.5, Ec=0.01

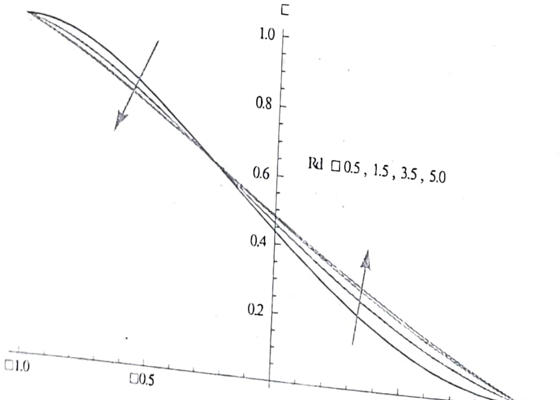

Fig.5c: Variation of θ with Rd
m=0.5, Rd=0.5, A11=0.5, B11=0.5, Ec=0.01

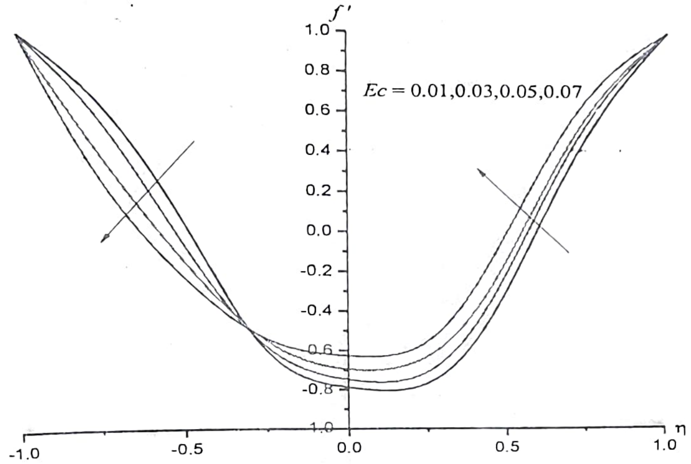

Fig.6a: Variation of f' with Ec
m=0.5, Rd=0.5, A11=0.5, B11=0.5, Ec=0.01

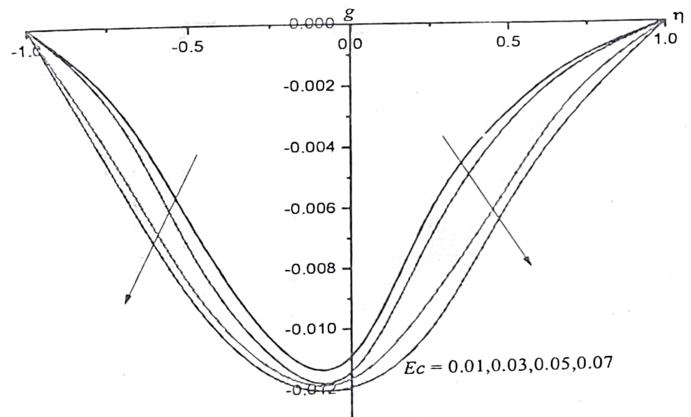

Fig.6b: Variation of *g* with Ec
m=0.5, Rd=0.5, A11=0.5, B11=0.5, Ec=0.01

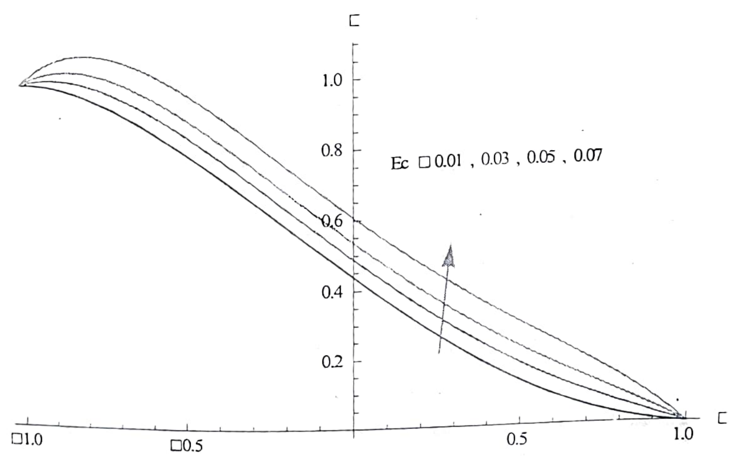

Fig.6c: Variation of θ with Ec
m=0.5, Rd=0.5, A11=0.5, B11=0.5

Table 1: Skin friction ($\tau_{x,z}$), Nusselt number at $\eta = \pm 1$

Parameter		τx(-1)	τx(+1)	τz(-1)	τz(+1)	Nu(-1)	Nu(+1)
m	0.5	-3.25666	2.95861	-0.000757626	-0.00116044	-0.029767	0.0488547
	1.0	-3.2515	2.94812	-0.00124599	-0.0011163	-0.0545257	0.0808118
	1.5	-3.2508	2.91608	-0.00199101	-0.000555654	-0.229862	0.260591
	2.0	-3.27818	2.87177	-0.00197867	-0.000375453	-0.644577	0.690337
Rd	0.5	-3.25666	2.95861	-0.000757626	-0.00116044	-0.029767	0.0488547
	1.5	-3.25793	2.94973	-0.000795749	-0.00119661	0.209449	0.240197
	3.5	-3.2594	2.94366	-0.000823797	-0.00122341	0.346184	0.359517
	5.0	-3.25991	2.94177	-0.000832658	-0.00123189	0.386331	0.395492
A11	0.5	-3.25666	2.95861	-0.000757626	-0.00116044	-0.029767	0.0488547
	1.5	-3.26043	2.96526	-0.000746207	-0.0011513	-0.312561	-0.240086
	-0.5	-3.23906	2.93373	-0.000788772	-0.00118242	1.13962	1.19851
	-1.5	-3.2348	2.92735	-0.000797643	-0.00118896	1.43006	1.48611
B11	0.5	-3.25666	2.95861	-0.000757626	-0.00116044	-0.029767	0.0488547
	1.5	-3.24211	2.97084	-0.000664326	-0.00106752	0.145395	-0.00640622
	-0.5	-3.35404	2.87125	-0.00140612	-0.00180257	-0.986485	0.503311
	-1.5	-3.40061	2.82755	-0.00172494	-0.00211625	-1.37771	0.749646
Ec	0.01	-3.25666	2.95861	-0.000757626	-0.00116044	-0.029767	0.0488547
	0.03	-3.27244	2.94281	-0.000864145	-0.00126587	-0.292323	0.291211
	0.05	-3.28727	2.92808	-0.000963983	-0.0013646	-0.540318	0.510564
	0.07	-3.30957	2.90597	-0.00111404	-0.0015128	-0.916583	0.836042

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